EXPONENTIAL SMOOTHING

Exponential smoothing gives less weight to older data values. Why? The updating equation applies to any time period, so:

$F_{t+1} = \alpha\, y_t + (1-\alpha)\, F_t$

$F_t = \alpha\, y_{t-1} + (1-\alpha)\, F_{t-1}$

$F_{t-1} = \alpha\, y_{t-2} + (1-\alpha)\, F_{t-2}$

$F_{t-2} = \alpha\, y_{t-3} + (1-\alpha)\, F_{t-3}$

etc.

Substituting, we get:

$F_{t+1} = \alpha\, y_t + (1-\alpha)\, [\alpha\, y_{t-1} + (1-\alpha)\, F_{t-1}]$ which simplifies to

$= \alpha\, y_t + (1-\alpha)\alpha\, y_{t-1} + (1-\alpha)^2\, F_{t-1}$

$= \alpha\, y_t + (1-\alpha)\alpha\, y_{t-1} + (1-\alpha)^2\, [\alpha\, y_{t-2} + (1-\alpha)\, F_{t-2}]$ which simplifies to

$= \alpha\, y_t + (1-\alpha)\alpha\, y_{t-1} + (1-\alpha)^2\, \alpha\, y_{t-2} + (1-\alpha)^3\, F_{t-2}$

$= \alpha\, y_t + (1-\alpha)\alpha\, y_{t-1} + (1-\alpha)^2\, \alpha\, y_{t-2} + (1-\alpha)^3\, [\alpha\, y_{t-3} + (1-\alpha)\, F_{t-3}]$ which simplifies to

$= \alpha\, y_t + (1-\alpha)\alpha\, y_{t-1} + (1-\alpha)^2\, \alpha\, y_{t-2} + (1-\alpha)^3\, \alpha\, y_{t-3} + (1-\alpha)^4\, F_{t-3}$

etc.

But if $0 \le \alpha \le 1$, then $(1-\alpha) > (1-\alpha)^2 > (1-\alpha)^3 > (1-\alpha)^4$. For example, if $\alpha = .2$ and $1-\alpha = .8$, then $.8 > .8^2 > .8^3 > .8^4$. How does the smoothing equation work for this example?

$F_{t+1} = .2\, y_t + .8\, F_t$

$F_t = .2\, y_{t-1} + .8\, F_{t-1}$

$F_{t-1} = .2\, y_{t-2} + .8\, F_{t-2}$

$F_{t-2} = .2\, y_{t-3} + .8\, F_{t-3}$

etc.

Substituting, we get:

$F_{t+1} = .2\, y_t + .8\, (.2\, y_{t-1} + .8\, F_{t-1})$ which simplifies to

$= 2\, y_t + .16\, y_{t-1} + .8^2\, F_{t-1}$

$= .2\, y_t + .16\, y_{t-1} + .8^2\, (.2\, y_{t-2} + .8\, F_{t-2})$ which simplifies to

$= .2\, y_t + .16\, y_{t-1} + .128\, y_{t-2} + .8^3\, F_{t-2}$

$= .2 y_t + .16\, y_{t-1} + .128\, y_{t-2} + .8^3\, (.2\, y_{t-3} + .8\, F_{t-3})$ which simplifies to

$= .2\, y_t + .16\, y_{t-1} + .128\, y_{t-2} + .1024\, y_{t-3} + (.8)^4\, F_{t-3}$

etc.

Thus, F_{t+1} is affected by all the past data values, but with diminishing weight.